Quiz for Week5-6

1. (20points) Matrix computation:

(1) (5points)

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^n$$

(2) (5points)

$$\begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}^n$$

(3) (10points) Give the inverse and transpose

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 2 \\ 3 & 3 & 3 \end{bmatrix}$$

2. (10points)

(1) How many 3 by 3 permutation matices are there? Please give them all. (Identity matrix included)

(2) Find a 4 by 4 permutation matrix P with $P^5 = P$

3. (10points) A is a square matrix.
(1) Prove that $X = (-A)A^T$ is a symmetric matrix.

(2) Tell whether XY is a symmetric matrix or not in which $Y = A^T A$

4. (20points) The matrix P that multiplies $(x_1, x_2, x_3, x_4)^T$ to give $(x_1, x_3, x_4, x_2)^T$ is also a rotation matrix.
(1) Find P and P^3.

(2) Give the rotation axis.

(3) What is the angle of rotation from $v = (4, 3, 5, -\sqrt{14})^T$ to $Pv = (4, 5, -\sqrt{14}, 3)^T$

(4) Give the angle between v and Pv

5. (10points) The sum of all the entries on the diagonal of a matrix A is called trace and written as $\mathrm{Tr}(A)$.

(1) Please prove that $\mathrm{Tr}(\boldsymbol{x}\boldsymbol{y}^{\boldsymbol{T}})$= $\mathrm{Tr}(\boldsymbol{y}^{\boldsymbol{T}}\boldsymbol{x})$ in which $\boldsymbol{x}$, $\boldsymbol{y}$ are both vectors.
(2) Please prove that $\mathrm{Tr}\boldsymbol{(AB)} = \mathrm{Tr}\boldsymbol{(BA)}$ when $\boldsymbol{BA}$ and $\boldsymbol{AB}$ both exists.