

## 20221020Assignment- Block Matrix

1.  $A$  and  $B$  are both invertible, please give  $D^{-1}$  for the  $D$  below:

$$D = \begin{bmatrix} A & O \\ C & B \end{bmatrix}$$

Does it matter whether  $C$  is invertible or not?

2. Elimination for a block matrix: When you multiply the first block row  $\begin{bmatrix} A & B \end{bmatrix}$  by  $CA^{-1}$  and subtract from the second row  $\begin{bmatrix} C & D \end{bmatrix}$ , the "Schur complement"  $S$  appears:

$$\begin{bmatrix} I & O \\ -CA^{-1} & I \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A & B \\ O & S \end{bmatrix}$$

- (1) Please use  $A, B, C, D$  to give  $S$

(2) If an identity matrix is wanted after some column operations applied to the matrix  $\begin{bmatrix} A & B \\ O & S \end{bmatrix}$ , can you give a matrix multiplication to denote this process? What matrix do you need? Should it be multiplied on the left or right?

- (3) If  $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 2 & 3 & 3 \\ 4 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix}$ , please find  $S$

3. Give the inverse of the following matrix  $A$ :

$$A = \begin{bmatrix} 1 & 1 & -1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 & 3 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & 1 \end{bmatrix}$$

- (1) by the Desmos or Excel;

- (2) by block matrix method.

Then check whether they are the same.