Quiz for week 4.

1\. (1) $\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 4 \end{pmatrix}$ row: 1st, $\times(-1)$, 1st & 2nd row exchange, 3rd row $\times 2$

(2) $\begin{pmatrix} 2 & -2 & 3 \\ 5 & -8 & 6 \\ 8 & -14 & 9 \end{pmatrix}$. 1st column $\times(-2)$, 1st & 2nd column exchange

(3) $\begin{pmatrix} 3 & 6 & 3 \\ 10 & -5 & 10 \\ -18 & -18 & -18 \end{pmatrix}$ row: 1st $\times 3$, 2nd $\times 5$, 3rd $\times(-6)$

2\. ① $E_{21} = \begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

② $E_{31} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$

③ $E_{32} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$.

3\. A must have no more than 2 pivots.

$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & a \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & -1 & a-9 \end{pmatrix}$.

$\to \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & a-7 \end{pmatrix}$. $\therefore a-7=0$, $a=7$

$\therefore \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$. $\to \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

$\therefore \begin{cases} x_1 - x_3 = b_1 \\ x_2 + 2x_3 = b_2 \\ 0 = b_3. \end{cases}$

$\therefore \begin{cases} x_1 = b_1 + k \\ x_2 = b_2 - 2k \\ x_3 = k \end{cases}$ $(k \in R)$.

4\. (1) $A^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

$\because AA = I$

$\therefore A^{-1} = A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

(2) $\left(\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ -1 & 2 & 0 & 1 \end{array}\right)$

$\to \left(\begin{array}{cc|cc} 1 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{5}{2} & \frac{1}{2} & 1 \end{array}\right)$.

$\to \left(\begin{array}{cc|cc} 1 & 0 & \frac{1}{2}-\frac{1}{10} & -\frac{1}{5} \\ 0 & 1 & \frac{1}{5} & \frac{2}{5} \end{array}\right)$

$\therefore A^{-1} = \begin{pmatrix} \frac{2}{5} & -\frac{1}{5} \\ \frac{1}{5} & \frac{2}{5} \end{pmatrix}$.

(3) $\left(\begin{array}{cccc|cccc} 0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & -3 & 0 & 0 & 1 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 & 1 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right)$

$\to \left(\begin{array}{cccc|cccc} 6 & 0 & 0 & 0 & 6 & 0 & 0 & 1 \\ 0 & 5 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -3 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 \end{array}\right)$.

$$\rightarrow\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & & & & \frac{1}{6} \\ 0 & 1 & 0 & 0 & & 0 & \frac{1}{5} & \\ 0 & 0 & 1 & 0 & & -\frac{1}{3} & & \\ 0 & 0 & 0 & 1 & \frac{1}{2} & & 0 & \end{array}\right).$$

$$A^2=\begin{pmatrix} 0 & 0 & 0 & 2 \\ 0 & 0 & -3 & 0 \\ 0 & 5 & 0 & 0 \\ 6 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 & 2 \\ 0 & 0 & -3 & 0 \\ 0 & 5 & 0 & 0 \\ 6 & 0 & 0 & 0 \end{pmatrix}$$

$$=\begin{pmatrix} 12 & 0 & 0 & 0 \\ 0 & -15 & 0 & 0 \\ 0 & 0 & -15 & 0 \\ 0 & 0 & 0 & 12 \end{pmatrix}$$

(4) A^{-1}:

$$\left(\begin{array}{ccc|ccc} 2 & 3 & 4 & 1 & 0 & 0 \\ 1 & 2 & -3 & 0 & 1 & 0 \\ 4 & 5 & 9 & 0 & 0 & 1 \end{array}\right).$$

$$\rightarrow\begin{pmatrix} -\frac{11}{3} & \frac{7}{9} & \frac{17}{9} \\ \frac{7}{3} & -\frac{2}{9} & -\frac{10}{9} \\ \frac{1}{3} & -\frac{2}{9} & -\frac{1}{9} \end{pmatrix}$$

(5) $A^3=\begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ $A^4=O$.

5.

(1) $\vec{\beta}=\begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{pmatrix}$ $\vec{\alpha}^T=(\alpha_1, \alpha_2 \cdots \alpha_n)$

$\therefore A=\begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{pmatrix}(\alpha_1 \cdots \alpha_n)$

$$=\begin{pmatrix} \beta_1\alpha_1 & \beta_1\alpha_2 & \cdots & \beta_1\alpha_n \\ \beta_2\alpha_1 & \beta_2\alpha_2 & \cdots & \beta_2\alpha_n \\ \cdots & \cdots & \cdots & \cdots \\ \beta_n\alpha_1 & \beta_n\alpha_2 & \cdots & \beta_n\alpha_n \end{pmatrix}.$$

After elimination, it's easy to find that.

$$A\rightarrow\begin{pmatrix} \beta_1\alpha_1 & \beta_1\alpha_2 & \cdots & \beta_1\alpha_n \\ 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 \end{pmatrix}.$$

(2) $\beta_1\alpha_1+\beta_2\alpha_2+\cdots+\beta_n\alpha_n=0$

$\therefore \vec{\beta}\cdot\vec{\alpha}=0$.

$\therefore \vec{\beta}\perp\vec{\alpha}$