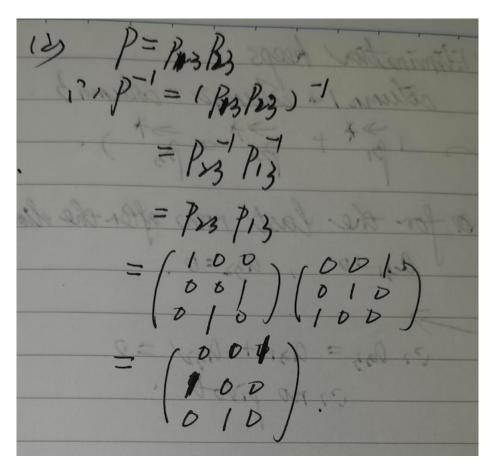
1
$$A^{-1} = \begin{bmatrix} 0 & \frac{1}{4} \\ \frac{1}{3} & 0 \end{bmatrix}$$
 and $B^{-1} = \begin{bmatrix} \frac{1}{2} & 0 \\ -1 & \frac{1}{2} \end{bmatrix}$ and $C^{-1} = \begin{bmatrix} 7 & -4 \\ -5 & 3 \end{bmatrix}$.

2 For the first, a simple row exchange has $P^2 = I$ so $P^{-1} = P$. For the second,

$$P^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$
. Always P^{-1} = "transpose" of P , coming in Section 2.7.



4 The equations are x+2y=1 and 3x+6y=0. No solution because 3 times equation 1 gives 3x+6y=3.

6.
$$AB-AC=D$$
 $A(B-C)=0$
 $Set B-C=M=(ab)$
 $(ab)=(ad)$
 $(ab)=(ad)$
 $(ab)=(ad)$
 $(ab)=(ad)$
 $(ab)=(ad)$
 $(ab)=(ad)$
 $(ab)=(ad)$
 $(ab)=(ad)$

- **6** (a) Multiply AB = AC by A^{-1} to find B = C (since A is invertible) (b) As long as B C has the form $\begin{bmatrix} x & y \\ -x & -y \end{bmatrix}$, we have AB = AC for $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.
- 7 (a) In Ax = (1,0,0), equation 1 + equation 2 equation 3 is 0 = 1 (b) Right sides must satisfy $b_1 + b_2 = b_3$ (c) Row 3 becomes a row of zeros—no third pivot.
- **8** (a) The vector $\mathbf{x} = (1, 1, -1)$ solves $A\mathbf{x} = \mathbf{0}$ (b) After elimination, columns 1 and 2 end in zeros. Then so does column 3 = column 1 + 2: no third pivot.
- **9** Yes, B is invertible (A was just multiplied by a permutation matrix P). If you exchange rows 1 and 2 of A to reach B, you exchange **columns** 1 and 2 of A^{-1} to reach B^{-1} . In matrix notation, B = PA has $B^{-1} = A^{-1}P^{-1} = A^{-1}P$ for this P.

8.
$$A = (\beta_1, \beta_2, \beta_3)$$
 $\overrightarrow{\chi} = \begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix}$

$$= (\beta_1, \beta_2, \beta_3) \begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix}$$

$$= (\beta_1, \beta_2, \beta_3) \begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \\ \chi_3 \end{pmatrix}$$

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$$= (\beta_1, \beta_2, \beta_3) \begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \\ \chi_3 \\ \chi_3 \end{pmatrix}$$

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$$= (\beta_1, \beta_2, \beta_3) \begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \\ \chi_3 \\ \chi_3 \end{pmatrix}$$

$$= (\beta_1, \beta_2, \beta_3) \begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \\ \chi_3 \\ \chi_3 \\ \chi_3 \end{pmatrix}$$

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$$= (\beta_1, \beta_2, \beta_3) \begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \\ \chi_3 \\ \chi_3 \end{pmatrix}$$

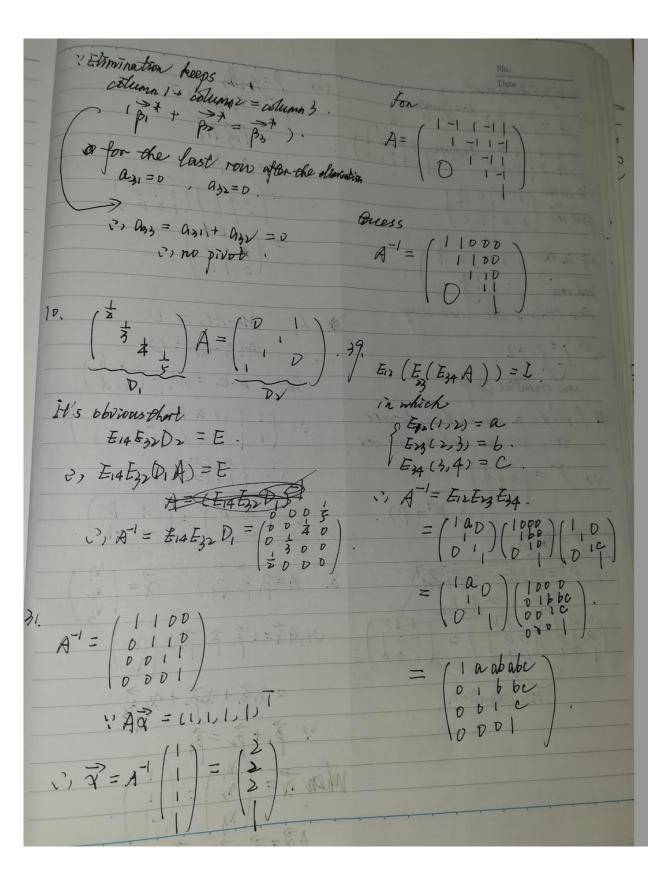
$$= (\beta_1, \beta_2, \beta_3) \begin{pmatrix} \chi_1 \\ \chi_3 \\ \chi_3 \\ \chi_3 \\ \chi_3 \end{pmatrix}$$

$$= (\beta_1, \beta_2, \beta_3) \begin{pmatrix} \chi_1 \\ \chi_3 \\ \chi_3 \\ \chi_3 \\ \chi_3 \\ \chi_3 \end{pmatrix}$$

$$= (\beta_1, \beta_2, \beta_3) \begin{pmatrix} \chi_1 \\ \chi_3 \end{pmatrix}$$

$$= (\beta_1, \beta_2, \beta_3) \begin{pmatrix} \chi_1 \\ \chi_3 \\ \chi_3$$

$$\mathbf{10} \ A^{-1} = \begin{bmatrix} 0 & 0 & 0 & 1/5 \\ 0 & 0 & 1/4 & 0 \\ 0 & 1/3 & 0 & 0 \\ 1/2 & 0 & 0 & 0 \end{bmatrix} \text{ and } B^{-1} = \begin{bmatrix} 3 & -2 & 0 & 0 \\ -4 & 3 & 0 & 0 \\ 0 & 0 & 6 & -5 \\ 0 & 0 & -7 & 6 \end{bmatrix} \text{ (invert each block of B)}$$



- **13** $M^{-1} = C^{-1}B^{-1}A^{-1}$ so multiply on the left by C and the right by $A: B^{-1} = CM^{-1}A$.
- **15** If A has a column of zeros, so does BA. Then BA = I is impossible. There is no A^{-1} .

- **29** (a) True (If A has a row of zeros, then every AB has too, and AB = I is impossible).
 - (b) False (the matrix of all ones is singular even with diagonal 1's.
 - (c) True (the inverse of A^{-1} is A and the inverse of A^2 is $(A^{-1})^2$).

30 Elimination produces the pivots
$$a$$
 and $a-b$ and $a-b$. $A^{-1} = \frac{1}{a(a-b)} \begin{bmatrix} a & 0-b \\ -a & a & 0 \\ 0-a & a \end{bmatrix}$.

The matrix C is not invertible if c = 0 or c = 7 or c = 2.

31
$$A^{-1} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
. When the triangular A has $1, -1, 1, -1, \ldots$ on successive

diagonals, A^{-1} is *bidiagonal* with 1's on the diagonal and first superdiagonal.

39 The inverse of
$$A = \begin{bmatrix} 1 & -a & 0 & 0 \\ 0 & 1 & -b & 0 \\ 0 & 0 & 1 & -c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
 is $A^{-1} = \begin{bmatrix} 1 & a & ab & abc \\ 0 & 1 & b & bc \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix}$. (This would

be a good example for the cofactor formula $A^{-1} = C^{\mathrm{T}}/\det A$ in Section 5.3)