1 $A^{-1} = \begin{bmatrix} 0 & \frac{1}{4} \\ \frac{1}{3} & 0 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} \frac{1}{2} & 0 \\ -1 & \frac{1}{2} \end{bmatrix}$ and $C^{-1} = \begin{bmatrix} 7 & -4 \\ -5 & 3 \end{bmatrix}$.

2 For the first, a simple row exchange has $P^2 = I$ so $P^{-1} = P$. For the second,

$P^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$. Always P^{-1} = "transpose" of P, coming in Section 2.7.

20220929 Assignment

1. $A = \begin{pmatrix} 0 & 3 \\ 4 & 0 \end{pmatrix} \xrightarrow{\text{Row Operation}} I$

$\left(\begin{array}{cc|cc} 0 & 3 & 1 & 0 \\ 4 & 0 & 0 & 1 \end{array}\right) \xrightarrow{\text{Exchange}} \left(\begin{array}{cc|cc} 4 & 0 & 0 & 1 \\ 0 & 3 & 1 & 0 \end{array}\right)$

$\xrightarrow[\text{first row}]{\times\frac{1}{4}\text{ in the}} \left(\begin{array}{cc|cc} 1 & 0 & 0 & \frac{1}{4} \\ 0 & 3 & 1 & 0 \end{array}\right)$

$\xrightarrow[\text{2nd row}]{\times\frac{1}{3}\text{ in the}} \left(\begin{array}{cc|cc} 1 & 0 & 0 & \frac{1}{4} \\ 0 & 1 & \frac{1}{3} & 0 \end{array}\right)$.

For others, the same way.

2. $\because$ $P^{-1}P = I$.

row operation to P

P needs a row exchange between the 1st row and the 3rd row

$\therefore P^{-1} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

$(\therefore P^{-1} = P)$.

(2) $P = P_{13}P_{23}$

$\therefore P^{-1} = (P_{13}P_{23})^{-1}$

$= P_{23}^{-1} P_{13}^{-1}$

$= P_{23} P_{13}$

$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

$= \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}.$

4 The equations are $x + 2y = 1$ and $3x + 6y = 0$. No solution because 3 times equation 1 gives $3x + 6y = 3$.

6: $AB - AC = 0$

$A(B-C) = 0$

Set $B - C = M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\therefore \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$\therefore \begin{cases} a + c = 0 \\ b + d = 0 \end{cases}$

$\therefore \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ -a & -b \end{pmatrix}.$

6 (a) Multiply $AB = AC$ by A^{-1} to find $B = C$ (since A is invertible) (b) As long as $B - C$ has the form $\begin{bmatrix} x & y \\ -x & -y \end{bmatrix}$, we have $AB = AC$ for $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

7 (a) In $A\boldsymbol{x} = (1, 0, 0)$, equation 1 + equation 2 − equation 3 is $0 = 1$ (b) Right sides must satisfy $b_1 + b_2 = b_3$ (c) Row 3 becomes a row of zeros—no third pivot.

8 (a) The vector $\boldsymbol{x} = (1, 1, -1)$ solves $A\boldsymbol{x} = \mathbf{0}$ (b) After elimination, columns 1 and 2 end in zeros. Then so does column 3 = column 1 + 2: no third pivot.

9 Yes, B is invertible (A was just multiplied by a permutation matrix P). If you exchange rows 1 and 2 of A to reach B, you exchange **columns** 1 and 2 of A^{-1} to reach B^{-1}. In matrix notation, $B = PA$ has $B^{-1} = A^{-1}P^{-1} = A^{-1}P$ for this P.

8. $A=(\vec{\beta}_1\ \vec{\beta}_2\ \vec{\beta}_3)$ $\quad \vec{x}=\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

$$\therefore A\vec{x}=(\vec{\beta}_1\ \vec{\beta}_2\ \vec{\beta}_3)\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$= x_1\vec{\beta}_1 + x_2\vec{\beta}_2 + x_3\vec{\beta}_3$$

$\because \vec{\beta}_1+\vec{\beta}_2=\vec{\beta}_3$

when $\vec{x}=\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$.

$$A\vec{x}=\vec{\beta}_1+\vec{\beta}_2-\vec{\beta}_3=0$$

10 $A^{-1}=\begin{bmatrix} 0 & 0 & 0 & 1/5 \\ 0 & 0 & 1/4 & 0 \\ 0 & 1/3 & 0 & 0 \\ 1/2 & 0 & 0 & 0 \end{bmatrix}$ and $B^{-1}=\begin{bmatrix} 3 & -2 & 0 & 0 \\ -4 & 3 & 0 & 0 \\ 0 & 0 & 6 & -5 \\ 0 & 0 & -7 & 6 \end{bmatrix}$ (invert each block of B)

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∵ Elimination keeps
column 1 + column 2 = column 3.
($\vec{\beta_1}^* + \vec{\beta_2}^* = \vec{\beta_3}^*$).

for the last row after the elimination
$a_{31} = 0$, $a_{32} = 0$.

∴ $a_{33} = a_{31} + a_{32} = 0$
∴ no pivot.

10.
$$\underbrace{\begin{pmatrix} \frac{1}{2} & & & \\ & \frac{1}{3} & & \\ & & \frac{1}{4} & \\ & & & \frac{1}{5} \end{pmatrix}}_{D_1} A = \underbrace{\begin{pmatrix} 0 & & & 1 \\ & & 1 & \\ & 1 & & \\ 1 & & & 0 \end{pmatrix}}_{D_2}.$$

It's obvious that
$E_{14}E_{32}D_2 = E$.

∴ $E_{14}E_{32}(D_1A) = E$

~~$A = (E_{14}E_{32}D_1)^{-1}$~~

∴ $A^{-1} = E_{14}E_{32}D_1 = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{5} \\ 0 & 0 & \frac{1}{4} & 0 \\ 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$.

31.
$$A^{-1} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

∵ $A\vec{x} = (1,1,1,1)^T$

∴ $\vec{x} = A^{-1}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 2 \\ 1 \end{pmatrix}$.

For
$$A = \begin{pmatrix} 1 & -1 & & \\ & 1 & -1 & \\ & & 1 & -1 \\ & & & 1 \end{pmatrix}$$

Guess
$$A^{-1} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ & 1 & 1 & 0 & 0 \\ & & 1 & 1 & 0 \\ & 0 & & 1 & 1 \\ & & & & 1 \end{pmatrix}$$

39.
$E_{12}(E_{23}(E_{34}A)) = I$.
in which
$E_{12}(1,2) = a$
$E_{23}(2,3) = b$.
$E_{34}(3,4) = c$.

∴ $A^{-1} = E_{12}E_{23}E_{34}$.

$= \begin{pmatrix} 1 & a & 0 & \\ & 1 & & \\ & & 1 & \\ 0 & & & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ & 1 & b & 0 \\ & & 1 & 0 \\ 0 & & & 1 \end{pmatrix}\begin{pmatrix} 1 & & & 0 \\ & 1 & & \\ & & 1 & c \\ 0 & & & 1 \end{pmatrix}$

$= \begin{pmatrix} 1 & a & 0 & \\ & 1 & & \\ & & 1 & \\ 0 & & & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & b & bc \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{pmatrix}$.

$= \begin{pmatrix} 1 & a & ab & abc \\ 0 & 1 & b & bc \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{pmatrix}$.

13 $M^{-1} = C^{-1}B^{-1}A^{-1}$ so multiply on the left by C and the right by A : $B^{-1} = CM^{-1}A$.

15 If A has a column of zeros, so does BA. Then $BA = I$ is impossible. There is no A^{-1}.

22 $$\begin{bmatrix} 1 & 3 & 1 & 0 \\ 2 & 7 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 3 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 7 & -3 \\ 0 & 1 & -2 & 1 \end{bmatrix} = \begin{bmatrix} I & A^{-1} \end{bmatrix};$$

$$\begin{bmatrix} 1 & 4 & 1 & 0 \\ 3 & 9 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 4 & 1 & 0 \\ 0 & -3 & -3 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -3 & 4/3 \\ 0 & 1 & 1 & -1/3 \end{bmatrix} = \begin{bmatrix} I & A^{-1} \end{bmatrix}.$$

29 (a) True (If A has a row of zeros, then every AB has too, and $AB = I$ is impossible).

(b) False (the matrix of all ones is singular even with diagonal 1's.

(c) True (the inverse of A^{-1} is A and the inverse of A^2 is $(A^{-1})^2$).

30 Elimination produces the pivots a and $a-b$ and $a-b$. $A^{-1} = \dfrac{1}{a(a-b)} \begin{bmatrix} a & 0 & -b \\ -a & a & 0 \\ 0 & -a & a \end{bmatrix}$.

The matrix C is not invertible if $c = 0$ or $c = 7$ or $c = 2$.

31 $A^{-1} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. When the triangular A has $1, -1, 1, -1, \ldots$ on successive diagonals, A^{-1} is *bidiagonal* with 1's on the diagonal and first superdiagonal.

39 The inverse of $A = \begin{bmatrix} 1 & -a & 0 & 0 \\ 0 & 1 & -b & 0 \\ 0 & 0 & 1 & -c \\ 0 & 0 & 0 & 1 \end{bmatrix}$ is $A^{-1} = \begin{bmatrix} 1 & a & ab & abc \\ 0 & 1 & b & bc \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix}$. (This would be a good example for the cofactor formula $A^{-1} = C^{\mathrm{T}}/\det A$ in Section 5.3)