Quiz for Week 1-3.

1. $\|\vec{v}\| = \sqrt{1^2+1^2+\cdots+1^2} = 3$ (2points)

∴ $\vec{u} = \frac{1}{\|\vec{v}\|}\vec{v}$

$= (\frac{1}{3}, \frac{1}{3}, \cdots \frac{1}{3})$. (2points)

$\vec{w} = (w_1, w_2, \cdots w_9)$

∵ $\begin{cases} \frac{1}{3}w_1 + \frac{1}{3}w_2 + \cdots + \frac{1}{3}w_9 = 0 & ① \\ w_1^2 + w_2^2 + \cdots + w_9^2 = 1. & ② \end{cases}$

From① (2points)

∴ $w_1 + w_2 + \cdots + w_9 = 0$

$w_9 = -(w_1 + \cdots + w_8)$

∴ Set $w_1 = \cdots = w_8 = a$

$w_9 = -8a$

∴ $w_1^2 + \cdots + w_9^2 = 1$

$a^2 + \cdots + a^2 + (-8a)^2 = 1$

$72a^2 = 1$

$a = \pm\sqrt{\frac{1}{72}}$

$= \pm\frac{\sqrt{2}}{12}$ (2points)

$-8a = \mp\frac{8}{12}\sqrt{2} = \mp\frac{2}{3}\sqrt{2}$

∴ $\vec{w}$ could be

$(\frac{\sqrt{2}}{12}, \frac{\sqrt{2}}{12}, \cdots, \frac{\sqrt{2}}{12}, -\frac{2}{3}\sqrt{2})$.

(2points).

2.

$2x-3y+z=1$ is parallel to the plane $2x-3y+z=0$

∴ Let's check the normal vector of $2x-3y+z=0$.

$\vec{v_0} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$ is a vector in the plane when $2x_0 - 3y_0 + z_0 = 0$.

Set the normal vector as

$\vec{n} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$.

∴ $\vec{n} \cdot \vec{v_0} = 0$

$ax_0 + by_0 + cz_0 = 0$

no matter what is the value of x_0, y_0, z_0.

∴ $a = 2$, $b = -3$, $c = 1$

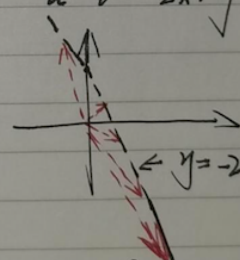
$\vec{n} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$.

3. $\vec{v} \cdot \vec{u} = \|\vec{v}\|\|\vec{u}\|\cos\theta$.

∴ $-\frac{1}{2} = 1 \times 1 \cos\theta$

$\cos\theta = -\frac{1}{2}$ ∴ $\theta = 120°$.

4. $\vec{u} \cdot \vec{v} = 2x + y = 5$



∴ the end point of $\vec{v}$ (x, y) always lies on the line l_0: $y = -2x+5$

∴ the shortest $\vec{v}$ should be perpendicular to the line l_0: $y = -2x+5$ or the vector $(1, -2) = \vec{w}$

∴ $\begin{cases} \vec{u} \cdot \vec{v} = 5 = 2x + y \\ \vec{w} \cdot \vec{v} = x - 2y = 0. \end{cases}$

∴ $\begin{cases} x = 2 \\ y = 1 \end{cases}$ ∴ the shortest $\vec{v}$ is $(2, 1)$.

5.

$$M\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y-x \\ x+2y-z \end{pmatrix}.$$

$$\therefore M = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & 2 & -1 \end{pmatrix}.$$

$$M\begin{pmatrix} x \\ y-x \\ x+2y-z \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}.$$

$$M = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 2 & -1 \end{pmatrix}.$$