6 $(A+B)^2 = \begin{bmatrix} 10 & 4 \\ 6 & 6 \end{bmatrix} = A^2 + AB + BA + B^2$. But $A^2 + 2AB + B^2 = \begin{bmatrix} 16 & 2 \\ 3 & 0 \end{bmatrix}$.

8 The rows of DA are 3 (row 1 of A) and 5 (row 2 of A). Both rows of EA are row 2 of A. The columns of AD are 3 (column 1 of A) and 5 (column 2 of A). The first column of AE is zero, the second is column 1 of A + column 2 of A.

11 Suppose EA does the row operation and then $(EA)F$ does the column operation (because F is multiplying from the right). The associative law says that $(EA)F = E(AF)$ so the column operation can be done first !

12 (a) $B = 4I$ (b) $B = 0$ (c) $B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ (d) Every row of B is $1, 0, 0$.

14 $(A-B)^2 = (B-A)^2 = A(A-B) - B(A-B) = A^2 - AB - BA + B^2$. In a typical case (when $AB \neq BA$) the matrix $A^2 - 2AB + B^2$ is different from $(A-B)^2$.

15 (a) True (A^2 is only defined when A is square).

(b) False (if A is m by n and B is n by m, then AB is m by m and BA is n by n).

(c) True by part (b).

(d) False (take $B = 0$).

19 Diagonal matrix, lower triangular, symmetric, all rows equal. Zero matrix fits all four.

21 $A^2 = \begin{bmatrix} 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, $A^3 = \begin{bmatrix} 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, $A^4 =$ zero matrix for *strictly triangular* A.

Then $A\boldsymbol{v} = A\begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 2y \\ 2z \\ 2t \\ 0 \end{bmatrix}$, $A^2\boldsymbol{v} = \begin{bmatrix} 4z \\ 4t \\ 0 \\ 0 \end{bmatrix}$, $A^3\boldsymbol{v} = \begin{bmatrix} 8t \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $A^4\boldsymbol{v} = \mathbf{0}$.

23 $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ has $A^2 = 0$. Note: Any matrix $A =$ column times row $= \boldsymbol{u}\boldsymbol{v}^{\mathrm{T}}$ will have $A^2 = \boldsymbol{u}\boldsymbol{v}^{\mathrm{T}}\boldsymbol{u}\boldsymbol{v}^{\mathrm{T}} = 0$ if $\boldsymbol{v}^{\mathrm{T}}\boldsymbol{u} = 0$. $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ has $A^2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ but $A^3 = 0$; strictly triangular as in Problem 21.

26 **Columns of A times rows of B** $\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}\begin{bmatrix} 3 & 3 & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 3 & 0 \\ 6 & 6 & 0 \\ 6 & 6 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 4 & 8 & 4 \\ 1 & 2 & 1 \end{bmatrix} =$

$\begin{bmatrix} 3 & 3 & 0 \\ 10 & 14 & 4 \\ 7 & 8 & 1 \end{bmatrix} = AB$.

32 A times $X = [\,\boldsymbol{x}_1 \;\; \boldsymbol{x}_2 \;\; \boldsymbol{x}_3\,]$ will be the identity matrix $I = [\,A\boldsymbol{x}_1 \;\; A\boldsymbol{x}_2 \;\; A\boldsymbol{x}_3\,]$.

38 (a) Multiply the columns $\boldsymbol{a}_1, \ldots, \boldsymbol{a}_m$ by the rows $\boldsymbol{a}_1^{\mathrm{T}}, \ldots, \boldsymbol{a}_m^{\mathrm{T}}$ and add the resulting matrices.