

7 To reverse E_{31} , **add** 7 times row 1 to row 3. The inverse of the elimination matrix

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -7 & 0 & 1 \end{bmatrix} \text{ is } E^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 7 & 0 & 1 \end{bmatrix}. \text{ Multiplication confirms } EE^{-1} = I.$$

9 $M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$. After the exchange, we need E_{31} (not E_{21}) to act on the new row 3.

12 The first product is $\begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$ rows and also columns reversed. The second product is $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -2 \\ 0 & 2 & -3 \end{bmatrix}$.

14 E_{21} has $-\ell_{21} = \frac{1}{2}$, E_{32} has $-\ell_{32} = \frac{2}{3}$, E_{43} has $-\ell_{43} = \frac{3}{4}$. Otherwise the E 's match I .

18 $EF = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix}$, $FE = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b+ac & c & 1 \end{bmatrix}$, $E^2 = \begin{bmatrix} 1 & 0 & 0 \\ 2a & 1 & 0 \\ 2b & 0 & 1 \end{bmatrix}$, $F^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3c & 1 \end{bmatrix}$.

19 $PQ = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$. In the opposite order, two row exchanges give $QP = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$,

$P^2 = I$. If M exchanges rows 2 and 3 then $M^2 = I$ (also $(-M)^2 = I$). There are

many square roots of I : Any matrix $M = \begin{bmatrix} a & b \\ c & -a \end{bmatrix}$ has $M^2 = I$ if $a^2 + bc = 1$.

- 22** (a) $\sum a_{3j}x_j$ (b) $a_{21} - a_{11}$ (c) $a_{21} - 2a_{11}$ (d) $(EA\mathbf{x})_1 = (A\mathbf{x})_1 = \sum a_{1j}x_j$.
- 23** $E(EA)$ subtracts 4 times row 1 from row 2 (EEA does the row operation twice).
 AE subtracts 2 times column 2 of A from column 1 (multiplication by E on the right side acts on **columns** instead of rows).
- 25** The last equation becomes $0 = 3$. If the original 6 is 3, then row 1 + row 2 = row 3.
Then the last equation is $0 = 0$ and the system has infinitely many solutions.
- 28** $A = AI = A(BC) = (AB)C = IC = C$. That middle equation is crucial.