No.

Date 2022.9.5 Assignment

① (a)

$\vec{v} = \begin{pmatrix}1\\2\\3\end{pmatrix}$. $\vec{w} = \begin{pmatrix}3\\6\\9\end{pmatrix}$.

$\therefore \vec{w} = 3\vec{v}$

for $\forall\ c, d$

$\vec{l} = c\vec{v} + d\vec{w}$

$= c\vec{v} + d\cdot 3\vec{v}$

$= (3d + c)\vec{v}$

$\therefore$ All the linear combinations of $\vec{v}, \vec{w}$ form a line where lies the vector $\vec{v}$.

(b). $\vec{v}, \vec{w}$ not in the same direct

$\therefore \vec{l} = c\vec{v} + d\vec{w}$ $(\forall c, d \in R)$.

All possibilities of $\vec{l}$ form a plane in R^3 where lies $\vec{v}$ and $\vec{w}$.

(c). $\vec{v} = \begin{pmatrix}2\\0\\0\end{pmatrix}$ $\vec{u} = \begin{pmatrix}0\\2\\2\end{pmatrix}$

$\vec{w} = \begin{pmatrix}2\\2\\3\end{pmatrix}$.

$\vec{l} = c\vec{v} + d\vec{u}$ form a plane where lies the $\vec{v}, \vec{w}$.

Check whether $\vec{w}$ is on the plane of $\vec{v}, \vec{u}$??

if $\vec{w}$ is on this plane

$\Rightarrow \vec{w} = c_1\vec{v} + d_1\vec{u}$.

$\begin{pmatrix}2\\2\\3\end{pmatrix} = c_1\begin{pmatrix}2\\0\\0\end{pmatrix} + d_1\begin{pmatrix}2\\2\\3\end{pmatrix}$.

When $\begin{cases} 2 = 2c_1 + 2d_1 \\ 2 = 0c_1 + 2d_1 \end{cases}$

Already have $\Rightarrow \begin{cases} c_1 = 0 \\ d_1 = 1 \end{cases}$

but in the 3rd row

$3 \neq 0\cdot c_1 + 3d_1$

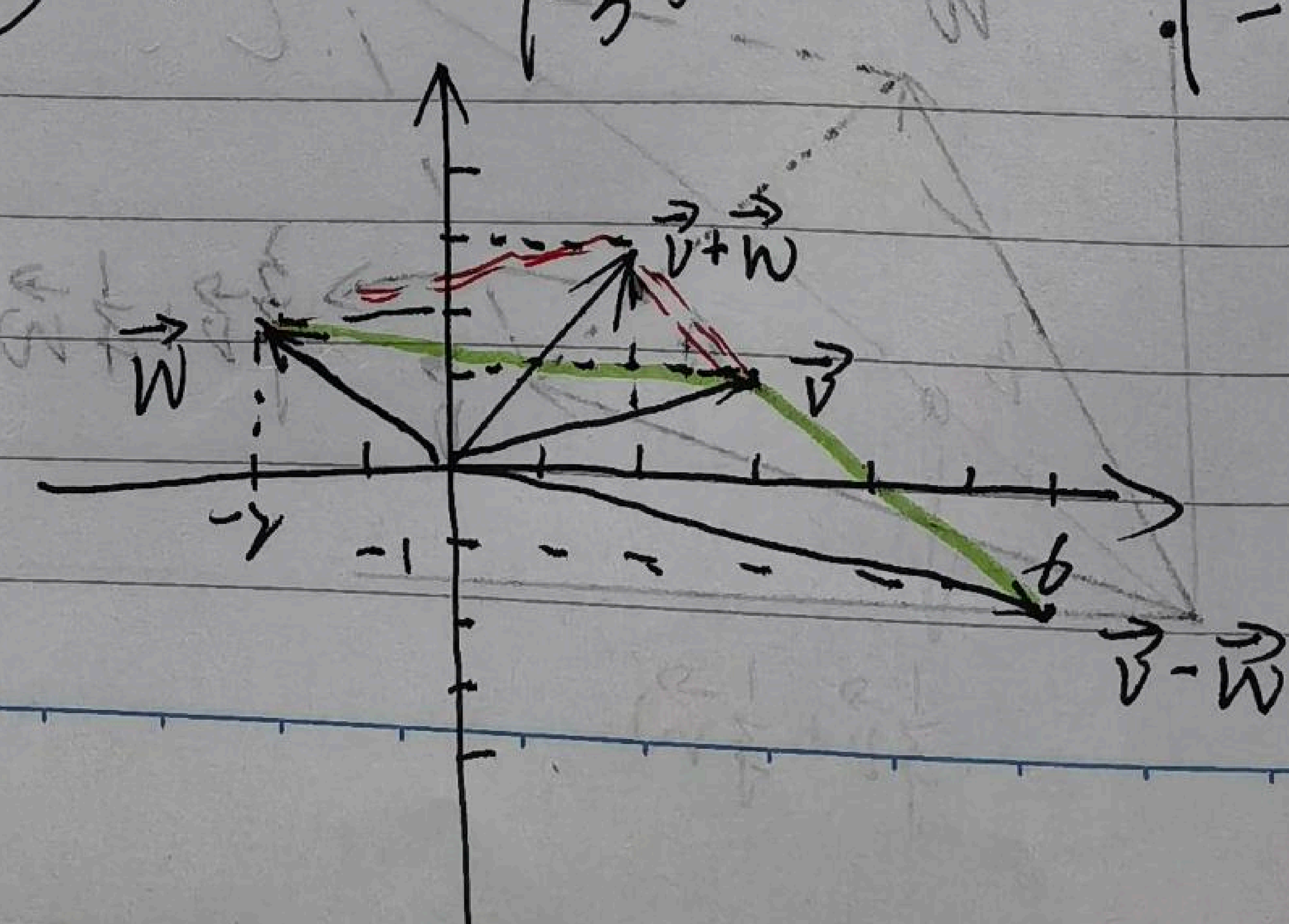
when $c_1 = 0$, $d_1 = 1$

$\therefore \vec{w} \neq c_1\vec{v} + d_1\vec{u}$.

$\therefore \vec{w}$ is not on this plane.

$\therefore \vec{n} = c\vec{v} + d\vec{u} + f\vec{w}$ form the whole R^3.

② $\vec{v} + \vec{w} = \begin{pmatrix}2\\3\end{pmatrix}$ $\vec{v} - \vec{w} = \begin{pmatrix}6\\-1\end{pmatrix}$



3. $2\vec{v} = (\vec{v}+\vec{w}) + (\vec{v}-\vec{w})$
$= \begin{pmatrix}6\\6\end{pmatrix}$

∴ $\vec{v} = \begin{pmatrix}3\\3\end{pmatrix}$

$\begin{pmatrix}3\\3\end{pmatrix} + \vec{w} = \begin{pmatrix}5\\1\end{pmatrix}$

$\vec{w} = \begin{pmatrix}2\\-2\end{pmatrix}$

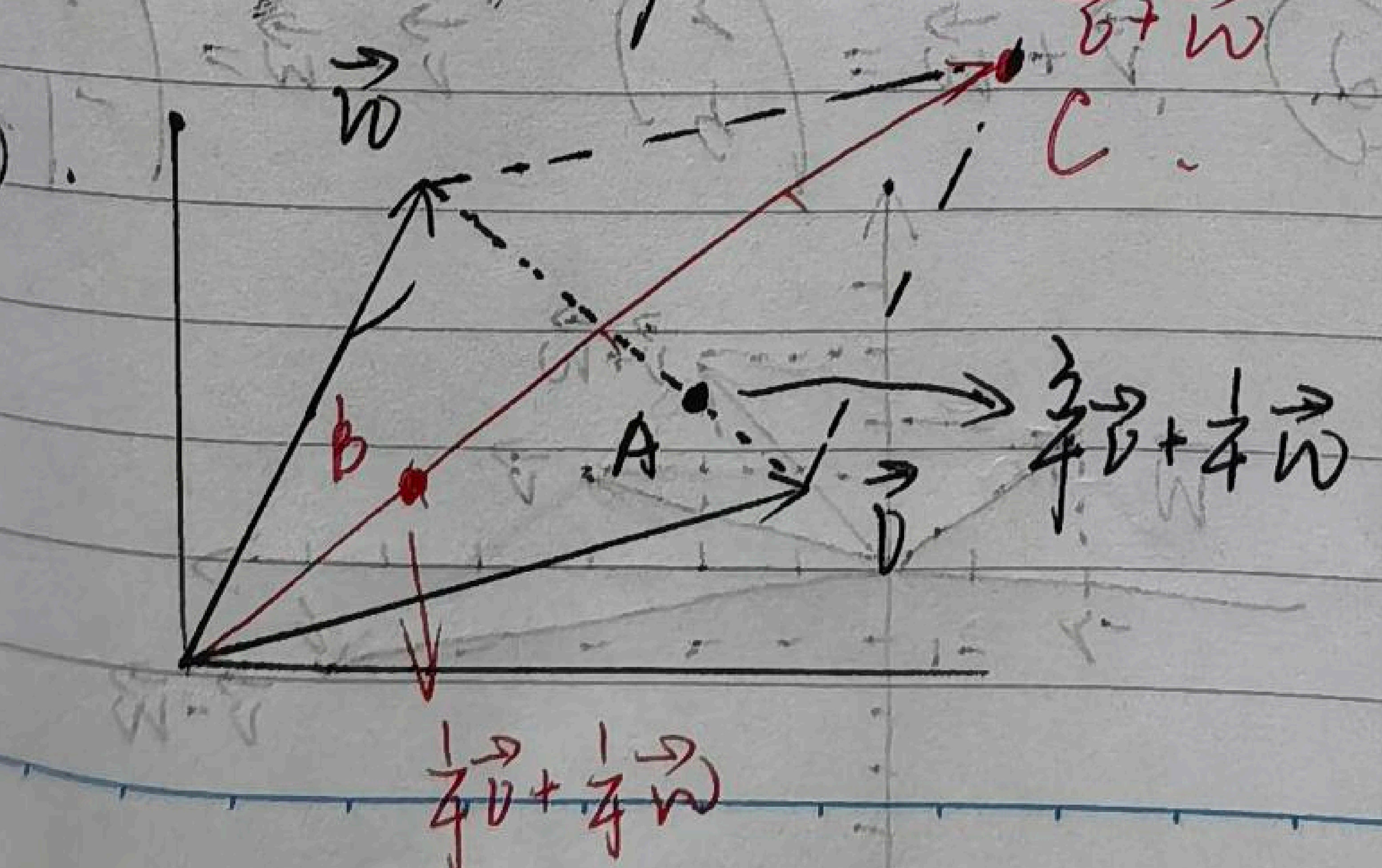
4. $3\vec{v}+\vec{w} = \begin{pmatrix}3\times 2+1\\3\times 1+2\end{pmatrix} = \begin{pmatrix}7\\5\end{pmatrix}$

$c\vec{v}+d\vec{w} = \begin{pmatrix}2c+d\\c+2d\end{pmatrix}$.

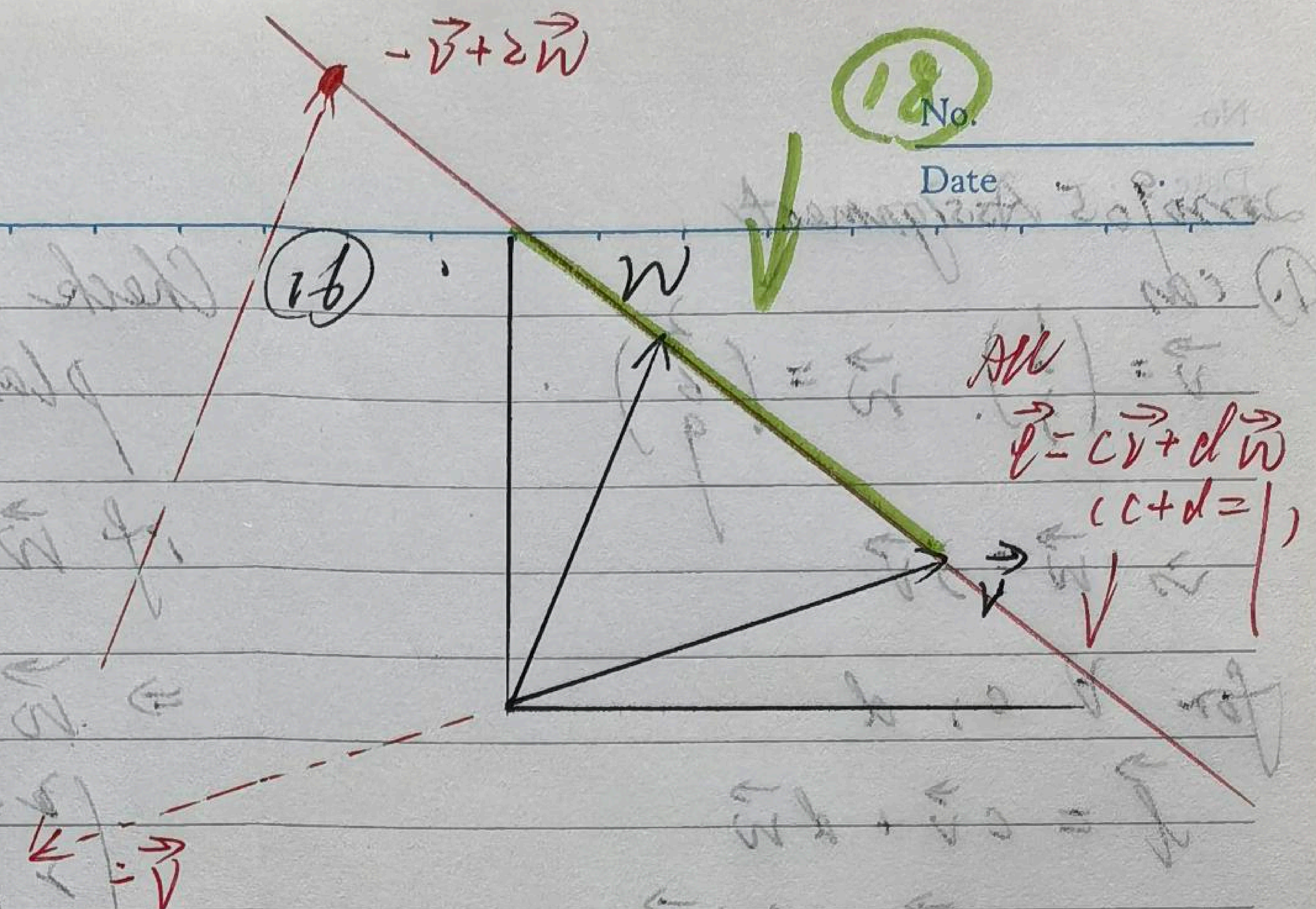
5. $\vec{u}+\vec{v}+\vec{w} = (0,0,0)^T$
$2\vec{u}+2\vec{v}+\vec{w} = (-2,3,1)^T$.
∵ $\vec{u}+\vec{v}+\vec{w} = (0,0,0)^T$
∴ They are in a same plane.

[if ∃ c, d, f.
s.t. $c\vec{u}+d\vec{v}+f\vec{w} = 0$.
⇒ $\vec{u}, \vec{v}, \vec{w}$ are in the same plane.

15.



16.



All $\vec{u} = c\vec{v}+d\vec{w}$
$(c+d=1)$

25. (a). $\vec{v} = k\vec{u} = \cancel{t}\vec{w}$
(b) $\vec{w} = k\vec{u} + t\vec{v}$

Optional

27. In R^3,
(0,0,0) will be connected to (1,0,0), (0,1,0), (0,0,1) for a typical cube.
R^3: one corner connects 3 edges.
∴ R^4: ...... 4 edges.

No. of corner:
(_ , _ , _):
↑ input 0 or 1.
∴ $2\times2\times2 = 8$
∴ for R^4, $2^4 = 16$.
∴ No. of edge: $\frac{16\times 4}{2} = 32$

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No. of faces.
take $(0,0,0,0)$ as a typical point.

$$\left\{\begin{array}{l}(0,0,0,0)\\(0,1,0,1)\\(0,1,0,0)\\(0,0,0,1)\end{array}\right.$$ form a plane.

How many choices for the point on the diagonal?

$(\_,\_,\_,\_)$.

$C_4^2 = 6$

$\therefore$ 6 faces/corner.

$\therefore \quad \frac{6\times 16}{4} = 24$.

20. (1) unless they lie on the same line

(2) $\vec{u} = (1,0,0,0)^T$

$\vec{v} = (0,1,0,0)^T$

$\vec{w} = (0,0,1,0)^T$.

$\vec{z} = (0,0,0,1)^T$