Mock Final

Math6002

- You have 90 minutes to finish all the tasks in real test
- Any plagiarism will lead to 0 score
- **Technology is allowed. But the process without tech is required for all tasks except for those noted by "Tech Allowed".**

1. (24points) $\boldsymbol{v_1} = (1,1,1)^T$, $\boldsymbol{v_2} = (2,0,-2)^T$, $\boldsymbol{b} = (2,3,4)^T$. Give
   a. the length of $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$
   b. the angle between $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$
   c. the equation for the plane S spanned by $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$
   d. the normal vector of S
   e. the orthogonal complement of S
   f. the projection of $\boldsymbol{b}$ onto the plane S
2. (6points) Are these vectors $\boldsymbol{v_1} = (1,1,1,1)^T$, $\boldsymbol{v_2} = (1,2,0,-2)^T$, $\boldsymbol{v_3} = (-3,0,6,8)^T$, and $\boldsymbol{v_4} = (0,6,-2,4)^T$ independent or dependent? Give the process.
3. (10points)

$$A = \begin{bmatrix} 1 & -2 & 2 \\ -2 & 6 & 10 \\ 3 & -5 & 2 \end{bmatrix}$$

   a. Give the inverse for A step by step
   b. (Tech Allowed) Solve the system of equation $A\boldsymbol{x} = \boldsymbol{b}$ in which $\boldsymbol{b} = (2,3,4)^T$
4. (10points) The nullspace of A is $x + 2y - z = 0$.
   a. Give an A
   b. Give the complete solution for $A\boldsymbol{x} = \boldsymbol{b}$ in which $\boldsymbol{b} = (4,8,-1)^T$
5. (10points) (Tech Allowed) $A = (\boldsymbol{v_1}, \boldsymbol{v_2})$, in which $v_1 = (0,1,2,\dots,100)^T$, $v_2 = \left(v_{2,1},\ v_{2,2},\dots,v_{2,100}\right)^T$, $b = \left(0^{\frac{1}{2}}, 1^{\frac{1}{2}},\dots,100^{\frac{1}{2}}\right)^T$
   a. If $v_{2,i} = i^2 + 1$ for any $i = 1,2,\dots,100$, give the least squares solution for $Ax = b$
   b. If $v_{2,i} = -3i$ for any $i = 1,2,\dots,100$, what will happen? Explain why will it happen.
6. (10points) For A

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

   a. Give the LU factorization
   b. Give the LDU factorization
7. (20points) Give the basis for (1)column space, (2)nullspace (3) row space, and (4) left nullspace of the following matrix A

$$A = \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 2 & 4 & 3 & 6 & 9 \\ 0 & 0 & 1 & -2 & -4 \end{bmatrix}$$

8. (5points) A, B are both square matrices and A is invertible. Prove that $rank(A) = rank(AB)$ if and only if B is invertible.
9. (5points) If $rank(\boldsymbol{\alpha_1}, \boldsymbol{\alpha_2}, \boldsymbol{\alpha_3}) = 3$, $rank(\boldsymbol{\alpha_1}, \boldsymbol{\alpha_2}, \boldsymbol{\alpha_3}, \boldsymbol{\alpha_4}) = 3$, $rank(\boldsymbol{\alpha_1}, \boldsymbol{\alpha_2}, \boldsymbol{\alpha_3}, \boldsymbol{\alpha_5}) = 4$, please prove that $rank(\boldsymbol{\alpha_1}, \boldsymbol{\alpha_2}, \boldsymbol{\alpha_3}, \boldsymbol{\alpha_5} - \boldsymbol{\alpha_4}) = 4$,