1. Prove that $C(AB) \subseteq C(A)$, in which $C(AB)$ is the column space of AB and $C(A)$ is A.

2. Give the column space of the following matrices:
   a. $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & -1 \\ 3 & 6 & 0 \end{bmatrix}$

   b. $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 1 \\ 3 & 2 & -1 \end{bmatrix}$

   c. $[A \;\; AB]$, in which $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 1 \\ 3 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$

3. Prove that $C(kA) = C(A)$ in which A is an m by n matrix, $k \neq 0$ is a real number, $C(kA)$ is the column space of kA and $C(A)$ is A

4. Prove that the column space of A is R^n if A is an n by n invertible matrix

5. Prove that the column space of $A_{m \times n}$ must be a subspace of R^k in which $k = \min(m, n)$