No.

Date

~~2021110~~ Assignment - Subspace.

9.

(a). $\{(x,y) \mid xy \in N\}$.

or $\{(x,y) \mid x,y \in Z\}$.

(b) $\{(x,y) \mid y=3x \text{ or } y=4x\}$

or

Remove x-axis from the plane but leave the origin

10. (a) √

It's a plane containing (0.0.0)

(b) X no origin

(c) X

$\vec{v}_1 = (0,1,2)$, $\vec{v}_2 = (1,0,2)$ both in it, but for $\vec{v}_1 + \vec{v}_2 = (1,1,4)$, it should be in the space but not.

(d). √ } planes containing (0.0.0)

(e). √

(f). X.

$\vec{v}_1 = (0,1,2)$ √

$-2\vec{v}_1 = (0,-2,-4)$ not in it.

11. (a)

$M_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ $M_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$.

∴ Smallest subspace

$S_M = \{\begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} \mid a,b \in R\}$.

(b) $S_M = \{\begin{pmatrix} a & a \\ 0 & 0 \end{pmatrix} \mid a \in R\}$

(c) $S_M = \{\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \mid a,b \in R\}$.

14. (a) R^2 itself.

All lines containing (0,0)

or $Z = \{(0,0)\}$

(b) $Z = \{\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}\}$.

or

$S_M = \{\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} \mid a \in R\}$.

or

$S_M = \{\begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} \mid a \in R\}$

or $S_M = \{\begin{pmatrix} 0 & 0 \\ 0 & a \end{pmatrix} \mid a \in R\}$.

or itself.

15. (a) line through (0,0,0).

(b) line through (0,0,0);

the point (0,0,0).

(c). for $S \cap T$.

if there is any $\vec{v}_1, \vec{v}_2$ in $S \cap T$.

~~there will be three situations~~

∵ $\vec{v}_1, \vec{v}_2$ both in S.

$\vec{v}_1, \vec{v}_2$ both in T.

∴ $\forall c_1\vec{v}_1$, $c_2\vec{v}_2$ in S & T. (∵ T, S are spaces)

$\forall \vec{v}_1 + \vec{v}_2$ in S & T (∵ S, T are spaces)

∴ $\forall c_1\vec{v}_1, c_2\vec{v}_2, \vec{v}_1+\vec{v}_2 \in S \cap T$.

16. P itself (L is in P)

or R^3 (L is not in P).

17. (a) if it is

$\mathbf{0} = 0 \cdot M_1$ should be in it

Zero Matrix

but $\mathbf{0}$ is not invertible

(b) For example, if $M_{2\times 2}$

$\begin{pmatrix}0&0\\0&1\end{pmatrix}$, $\begin{pmatrix}1&0\\0&0\end{pmatrix}$ ~~are~~ singular.

if it is a space, ∴

$\forall\ c_1\begin{pmatrix}0&0\\0&1\end{pmatrix} + c_2\begin{pmatrix}1&0\\0&0\end{pmatrix}$ should be in it,

but when $c_1 = 1$, $c_2 = 1$.

$\begin{pmatrix}1&0\\0&1\end{pmatrix}$ is an invertible matrix not in it,

18. (a) ✓

$\forall A_1, A_2 \in M$.

∵ $A_1 = A_1^T$ $\quad A_2 = A_2^T$

∴ $A_1 + A_2 = A_1^T + A_2^T$
$= (A_1 + A_2)^T$

∴ $A_1 + A_2$ is in M.

$\forall\ c_1 A_1 = c_1 A_1^T$

∴ $c_1 A_1$ is in M.

(b) $\forall A_1, A_2 \in M$, ✓

$A_1 + A_2 = (-A_1^T) + (-A_2^T)$
$= -(A_1^T + A_2^T)$
$= -(A_1 + A_2)^T$

∴ $A_1 + A_2$ in the M.

also, $\forall\ c_1 A_1$ in the M,

(c) ✗ $\begin{pmatrix}1&0\\0&0\end{pmatrix} + \begin{pmatrix}0&0\\0&1\end{pmatrix} = \underbrace{\begin{pmatrix}1&0\\0&1\end{pmatrix}}$

should be in M, but not.

30. a) If $\vec{u}, \vec{v}$ are both in S+T.

∴ $\vec{u} = \vec{s}_1 + \vec{t}_1$ $\quad$ ($\vec{s}_1, \vec{s}_2 \in S$,

$\vec{v} = \vec{s}_2 + \vec{t}_2$ $\quad$ $\vec{t}_1, \vec{t}_2 \in T$).

① ∴ $\vec{u} + \vec{v} = (\vec{s}_1 + \vec{s}_2) + (\vec{t}_1 + \vec{t}_2)$

∵ $\vec{s}_1 + \vec{s}_2 \in S$

$\vec{t}_1 + \vec{t}_2 \in T$

∴ $\vec{u} + \vec{v} \in S + T$.

② for $c\vec{u} = c\vec{s}_1 + c\vec{t}_1$

∵ $c\vec{s}_1 \in S$ $\quad c\vec{t}_1 \in T$

∴ $c\vec{u} \in S + T$.

b). ~~S+T~~ S+T is a plane spanned by these two lines.

~~S+T~~

S∪T is just the two lines.

∵ S∪T is not a space.

∴ A smallest subspace containing S∪T should be spanned from S∪T.

And the result is just the plane containing the two lines, which is S+T.