Text for Unit1-2

1. (45points) Computation:

(1) (5points) Give the measure of the angle between vector $\beta = (1, 1, 1)^T$ and $\alpha = (1, -1, 1)^T$

(2) (5points) Give the inverse

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 3 \\ 1 & 3 & 2 \end{bmatrix}$$

(3) (5points)

$$\begin{bmatrix} 0 & 2 & 1 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix}^3$$

(4) (5points)

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 3 & 5 \\ 1 & 2 & 1 \\ 3 & 3 & -1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix}$$

(5) (5points) If $x = (x_1, x_2, ..., x_n)^T$, please give $x^T x$ and xx^T

(6) (10points) Give the LU and LDU factorization for A

$$A = \begin{bmatrix} 1 & 0 & 1 \\ -2 & -2 & -2 \\ 3 & 4 & 5 \end{bmatrix}$$

(7) (10points) $c_1 v + c_2 w + c_3 u = (1, 2, 3)^T$ in which v, w, u are all vectors and c_1, c_2, c_3 are all coefficients. $v = (1, 1, 1)^T$, $w = (1, 2, 1)$

a) If $u = (-1, -1, 2)$, give the value of c_1, c_2, c_3
b) If there is no solution for c_1, c_2, c_3, please give all possibilities for u

2. (25points)

(1) Find a 4 by 4 permutation matrix P with $P^3 = P$ in which $P \neq I$

(2) Prove that $A^T BA$ is a symmetric matrix if $B(i, j) = B(j, i)$ is always true for $\forall i, j \in \{1, 2, ..., n\}$ in which B is a $n \times n$ matrix

(3) Explain why $AB = BA(A, B$ are both matrices) is not correct for most of the cases and give 2 examples in which $AB = BA$ is true.

(4) Prove that $(AB)C = A(BC)$ in which A, B, C are all matrices.

(5) Prove that $P^{-1} = P^T$ is always true for any permutation matrix P